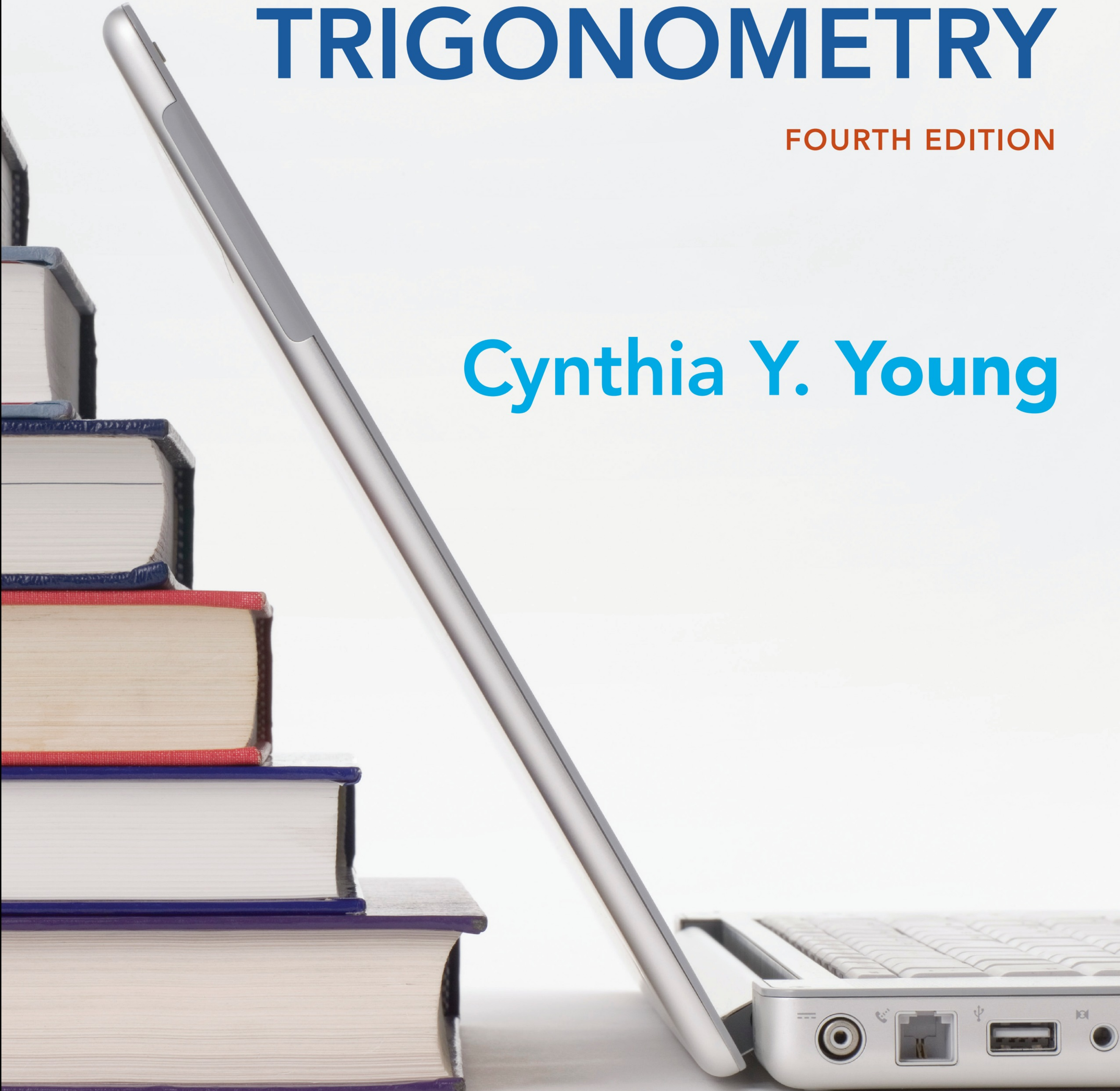
ALGEBRA AND TRIGONOMETRY

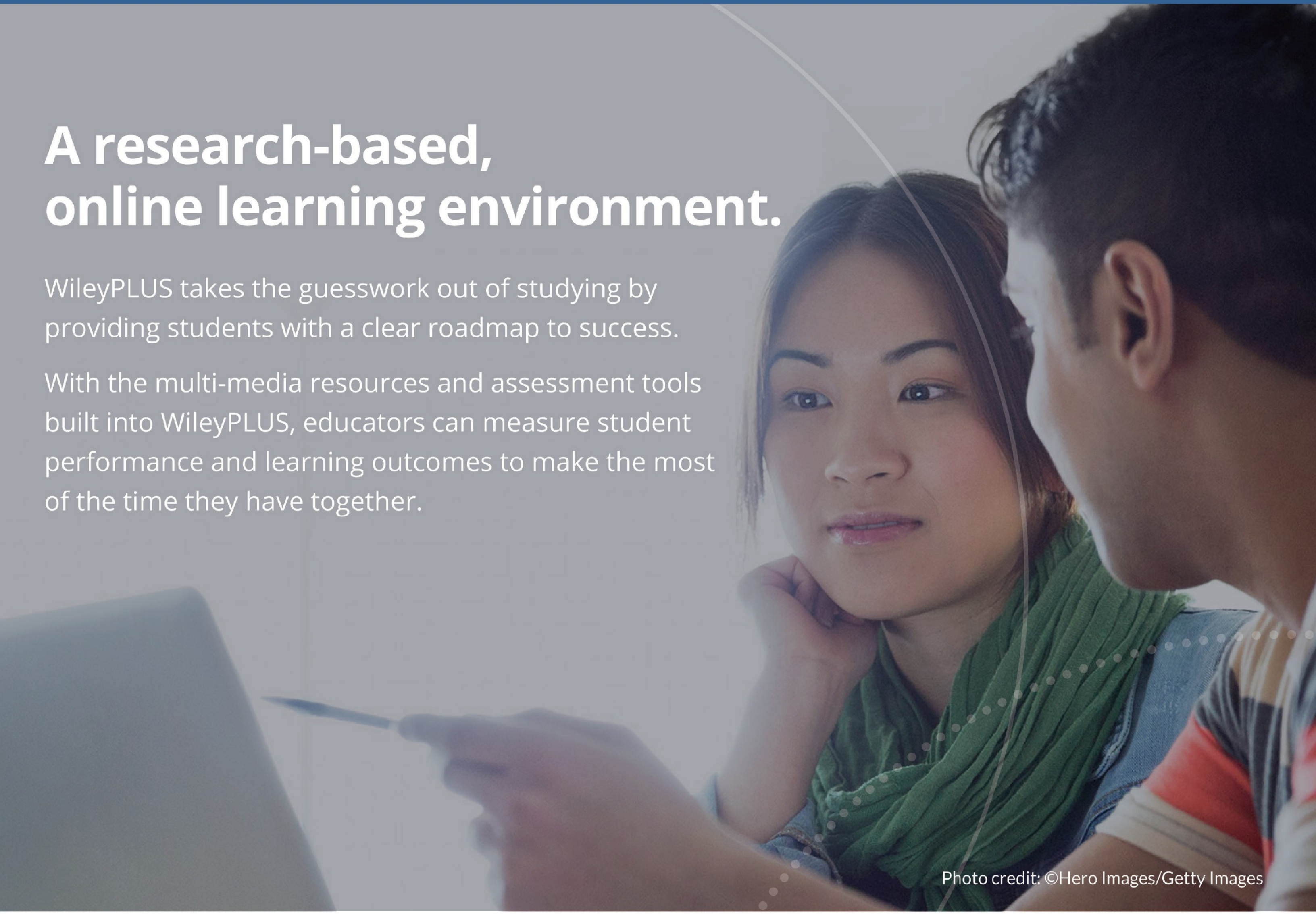
FOURTH EDITION

Cynthia Y. Young



WILEY

WileyPLUS



A research-based, online learning environment.

WileyPLUS takes the guesswork out of studying by providing students with a clear roadmap to success.

With the multi-media resources and assessment tools built into WileyPLUS, educators can measure student performance and learning outcomes to make the most of the time they have together.

Photo credit: ©Hero Images/Getty Images

Identify which students are struggling early in the semester.

Educators can track the students' progress through required or practice assignments, and know which students are keeping up with the pace of the course.

Help students organize their learning and get the practice they need.

The Read, Study & Practice area, organized by learning objective, provides students with the complete e-textbook and a wealth of resources for learning the material.

Measure outcomes to promote continuous improvement.

With Gradebook and learning objective-based results, both instructors and students can gauge problem areas and act on what's most important.

Algebra and Trigonometry

FOURTH EDITION

Cynthia Y. Young

PROFESSOR OF MATHEMATICS

University of Central Florida

WILEY

VICE PRESIDENT & DIRECTOR Laurie Rosatone
ACQUISITIONS EDITOR Joanna Dingle
ASSISTANT DEVELOPMENT EDITOR Ryann Dannelly
EDITORIAL ASSISTANT Giana Milazzo
MARKETING MANAGER John LaVacca III
SENIOR PRODUCT DESIGNER David Dietz
PRODUCT DESIGN MANAGER Thomas Kulesa
FREELANCE PROJECT EDITOR Anne Scanlan-Rohrer
PRODUCTION SERVICES Cenveo® Publisher Services
SENIOR CONTENT MANAGER Valerie Zaborski
SENIOR PRODUCTION EDITOR Ken Santor
SENIOR DESIGNER Maureen Eide
SENIOR PHOTO EDITOR Billy Ray
COVER PHOTO Jupiter Images/Getty Images

This book was set in 10/12 Times by Cenveo® Publisher Services, and printed and bound by Quad/Graphics, Inc.

The cover was printed by Quad/Graphics, Inc.

This book is printed on acid free paper.

Founded in 1807, John Wiley & Sons, Inc. has been a valued source of knowledge and understanding for more than 200 years, helping people around the world meet their needs and fulfill their aspirations. Our company is built on a foundation of principles that include responsibility to the communities we serve and where we live and work. In 2008, we launched a Corporate Citizenship Initiative, a global effort to address the environmental, social, economic, and ethical challenges we face in our business. Among the issues we are addressing are carbon impact, paper specifications and procurement, ethical conduct within our business and among our vendors, and community and charitable support. For more information, please visit our website: www.wiley.com/go/citizenship.

Copyright © 2017, 2013, 2009 John Wiley & Sons, Inc. All rights reserved.

No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning or otherwise, except as permitted under Sections 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, website www.copyright.com. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030-5774, (201) 748-6011, fax (201) 748-6008, website www.wiley.com/go/permissions.

Evaluation copies are provided to qualified academics and professionals for review purposes only, for use in their courses during the next academic year. These copies are licensed and may not be sold or transferred to a third party. Upon completion of the review period, please return the evaluation copy to Wiley. Return instructions and a free of charge return mailing label are available at www.wiley.com/go/returnlabel. If you have chosen to adopt this textbook for use in your course, please accept this book as your complimentary desk copy. Outside of the United States, please contact your local sales representative.

The inside back cover will contain printing identification and country of origin if omitted from this page. In addition, if the ISBN on the back cover differs from the ISBN on this page, the one on the back cover is correct.

ISBN: 978-1-119-32086-9 (ePub)
ISBN: 978-1-119-27345-5 (LLPC)

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

FOR

Christopher and Caroline

About the Author

University of Central Florida

University of Central Florida

Cynthia Y. Young is the Pegasus Professor of Mathematics and the Vice Provost for Faculty Excellence and UCF Global at the University of Central Florida (UCF) and the author of *College Algebra, Trigonometry, Algebra and Trigonometry,* and *Precalculus*. She holds a BA in Secondary Mathematics Education from the University of North Carolina (Chapel Hill), an MS in Mathematical Sciences from UCF, and both an MS in Electrical Engineering and a PhD in Applied Mathematics from the University of Washington. She has taught high school in North Carolina and Florida, developmental mathematics at Shoreline Community College in Washington, and undergraduate and graduate students at UCF.

Dr. Young joined the faculty at UCF in 1997 as an assistant professor of mathematics, and her primary research area was the mathematical modeling of the atmospheric effects on propagating laser beams. Her atmospheric propagation research was recognized by the Office of Naval Research Young Investigator Award, and in 2017 she was selected as a fellow of the International Society for Optical Engineering. Her secondary area of research centers on improvement of student learning in mathematics. She has authored or co-authored over 60 books and articles and has served as the principal investigator or co-principal investigator on projects with more than $2.5 million in federal funding. Dr. Young was on the team at UCF that developed the UCF EXCEL program, which was originally funded by the National Science Foundation to support the increase in the number of students graduating with a degree in science, technology, engineering, and mathematics (STEM). The EXCEL learning community approach centered around core mathematics courses has resulted in a significant increase in STEM graduation rates and has been institutionalized at UCF.

Dr. Young has been the recipient of many of UCF's awards (Excellence in Undergraduate Teaching, Excellence in Research, Teaching Incentive Program, Research Incentive Program, Scholarship of Teaching and Learning award, and UCF's highest honor, UCF Pegasus Professor). She has shared her techniques and experiences with colleagues around the country through talks at colleges, universities, and conferences.

Preface

As a mathematics professor, I would hear my students say, "I understand you in class, but when I get home I am lost." When I would probe further, students would continue with "I can't read the book." As a mathematician, I always found mathematics textbooks quite easy to read—and then it dawned on me: Don't look at this book through a mathematician's eyes; look at it through the eyes of students who might not view mathematics the same way that I do. What I found was that the books were not at all like my class. Students understood me in class, but when they got home they couldn't understand the book.

It was then that the folks at Wiley lured me into writing. My goal was to write a book that is seamless with how we teach and is an ally (not an adversary) to student learning. I wanted to give students a book they could read without sacrificing the rigor needed for conceptual understanding. The following quote comes from a reviewer when asked about the rigor of the book:

> *I would say that this text comes across as a little less rigorous than other texts, but I think that stems from how easy it is to read and how clear the author is. When one actually looks closely at the material, the level of rigor is high.*

DISTINGUISHING FEATURES

Four key features distinguish this book from others, and they came directly from my classroom.

PARALLEL WORDS AND MATH

Have you ever looked at your students' notes? I found that my students were only scribbling down the mathematics that I would write—never the words that I would say in class. I started passing out handouts that had two columns: one column for math and one column for words. Each example would have one or the other; either the words were there and students had to fill in the math or the math was there and students had to fill in the words. If you look at the examples in this book, you will see that the words (your voice) are on the left and the mathematics is on the right. In most math books, when the author illustrates an example, the mathematics is usually down the center of the page, and if the students don't know what mathematical operation was performed, they will look to the right for some brief statement of help. That's not how we teach; we don't write out an example on the board and then say, "Class, guess what I just did!" Instead we lead our students, telling them what step is coming and then performing that mathematical step *together*—and reading naturally from left to right. Student reviewers have said that the examples in this book are easy to read; that's because *your* voice is right there with them, working through problems *together*.

EXAMPLE 10 **Determining the Domain of a Function**

State the domain of the given functions.

a. $F(x) = \dfrac{3}{x^2 - 25}$ **b.** $H(x) = \sqrt[4]{9 - 2x}$ **c.** $G(x) = \sqrt[3]{x - 1}$

Solution (a):

Write the original equation.	$F(x) = \dfrac{3}{x^2 - 25}$
Determine any restrictions on the values of x.	$x^2 - 25 \neq 0$
Solve the restriction equation.	$x^2 \neq 25$ or $x \neq \pm\sqrt{25} = \pm 5$

SKILLS AND CONCEPTS (LEARNING OBJECTIVES AND EXERCISES)

In my experience as a mathematics teacher/instructor/professor, I find skills to be on the micro level and concepts on the macro level of understanding mathematics. I believe that too often skills are emphasized at the expense of conceptual understanding. I have purposely separated *learning objectives* at the beginning of every section into two categories: *skills objectives*—what students should be able to do—and *conceptual objectives*—what students should understand. At the beginning of every class, I discuss the learning objectives for the day—both skills and concepts. These are reinforced with both skills exercises and conceptual exercises. Each subsection has a corresponding skills objective and conceptual objective.

3.1 FUNCTIONS

SKILLS OBJECTIVES

- Determine whether a relation is a function.
- Determine whether an equation represents a function.
- Use function notation to evaluate functions for particular arguments.
- Determine the domain and range of a function.

CONCEPTUAL OBJECTIVES

- Understand that all functions are relations but not all relations are functions.
- Understand why the vertical line test determines if a relation is a function.
- Think of function notation as a placeholder or mapping.
- Understand the difference between implicit domain and explicit domain.

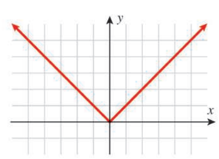
CATCH THE MISTAKE

Have you ever made a mistake (or had a student bring you his or her homework with a mistake) and you've gone over it and over it and couldn't find the mistake? It's often easier to simply take out a new sheet of paper and solve it from scratch than it is to actually find the mistake. Finding the mistake demonstrates a higher level of understanding. I include a few *Catch the Mistake* exercises in each section that demonstrate a common mistake. Using these in class (with individuals or groups) leads to student discussion and offers an opportunity for formative assessment in real time.

• CATCH THE MISTAKE

In Exercises 121–126, explain the mistake that is made.

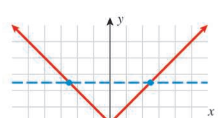
121. Determine whether the relationship is a function.



Solution:

Apply the horizontal line test.

Because the horizontal line intersects the graph in two places, this is not a function.



124. Determine the domain of the function $g(t) = \sqrt{3 - t}$ and express it in interval notation.

Solution:

What can t be? Any nonnegative real number.

$3 - t > 0$

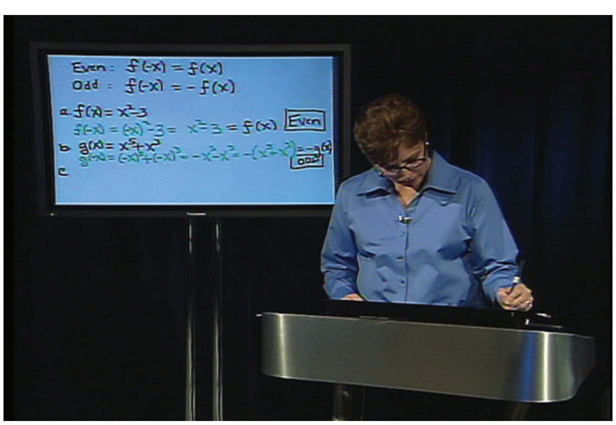
$3 > t$ or $t < 3$

Domain: $(-\infty, 3)$

This is incorrect. What mistake was made?

125. Given the function $G(x) = x^2$, evaluate $\dfrac{G(-1 + h) - G(-1)}{h}$.

Solution:



LECTURE VIDEOS BY THE AUTHOR

I authored the videos to ensure consistency in the students' learning experience. Throughout the book, wherever a student sees the video icon, that indicates a video. These videos provide mini lectures. The chapter openers and chapter summaries act as class discussions. The "Your Turn" problems throughout the book challenge the students to attempt a problem similar to a nearby example. The "worked-out example" videos are intended to come to the rescue for students if they get lost as they read the text and work problems outside the classroom.

NEW TO THE FOURTH EDITION

In the fourth edition, the main upgrades are updated applications throughout the text; new Skills and Conceptual objectives mapped to each subsection; new Concept Check questions in each subsection; and the substantially improved version of *WileyPLUS*, including ORION adaptive practice and interactive animations.

SKILLS AND CONCEPTUAL OBJECTIVES

3.1 FUNCTIONS

SKILLS OBJECTIVES

- Determine whether a relation is a function.
- Determine whether an equation represents a function.
- Use function notation to evaluate functions for particular arguments.
- Determine the domain and range of a function.

CONCEPTUAL OBJECTIVES

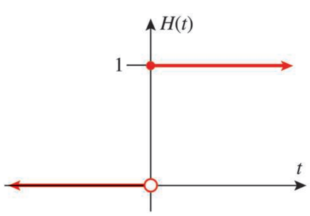
- Understand that all functions are relations but not all relations are functions.
- Understand why the vertical line test determines if a relation is a function.
- Think of function notation as a placeholder or mapping.
- Understand the difference between implicit domain and explicit domain.

CONCEPT CHECK QUESTIONS

[CONCEPT CHECK]

If the domain consists of all physical (home) addresses in a particular county and the range is the persons living in that county, does this describe a relation? And if so, is that relation a function?

ANSWER This is a relation but not a function.



[CONCEPT CHECK]

State the domain, range, and any points of discontinuity for the Heaviside function.

ANSWER Domain: $(-\infty, \infty)$
Range: $[0] \cup [1]$
Point of Discontinuity: $x = 0$

[CONCEPT CHECK]

Given a fixed distance, the time it takes you to drive that distance varies inversely with ________?

ANSWER rate (or speed)

APPLICATIONS TO BUSINESS, ECONOMICS, HEATH SCIENCES, AND MEDICINE

• APPLICATIONS

101. Budget: Event Planning. The cost associated with a catered wedding reception is \$45 per person for a reception for more than 75 people. Write the cost of the reception in terms of the number of guests and state any domain restrictions.

102. Budget: Long-Distance Calling. The cost of a local home phone plan is \$35 for basic service and \$0.10 per minute for any domestic long-distance calls. Write the cost of monthly phone service in terms of the number of monthly long-distance minutes and state any domain restrictions.

103. Temperature. The average temperature in Tampa, Florida, in the springtime is given by the function $T(x) = -0.7x^2 + 16.8x - 10.8$, where T is the temperature in degrees Fahrenheit and x is the time of day in military time and is restricted to $6 \le x \le 18$ (sunrise to sunset). What is the temperature at 6 A.M.? What is the temperature at noon?

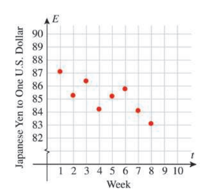
104. Falling Objects: Firecrackers. A firecracker is launched straight up, and its height is a function of time, $h(t) = -16t^2 + 128t$, where h is the height in feet and t is the time in seconds with $t = 0$ corresponding to the instant it launches. What is the height 4 seconds after launch? What is the domain of this function?

105. Collectibles. The price of a signed Alex Rodriguez baseball card is a function of how many are for sale. When Rodriguez was traded from the Texas Rangers to the New York Yankees in 2004, the going rate for a signed baseball

108. Volume. A cylindrical water basin will be built to harvest rainwater. The basin is limited in that the largest radius it can have is 10 feet. Write a function representing the volume of water V as a function of height h. How many additional gallons of water will be collected if you increase the height by 2 feet? *Hint:* 1 cubic foot = 7.48 gallons.

For Exercises 109–110, refer to the following:

The weekly exchange rate of the U.S. dollar to the Japanese yen is shown in the graph as varying over an 8-week period. Assume the exchange rate $E(t)$ is a function of time (week); let $E(1)$ be the exchange rate during Week 1.



114. Environment: Tossing the Envelopes. Each month, Jack receives his bank statement in a 9.5 inch by 6 inch envelope. Each month, he throws away the envelope after removing the statement.

a. The width of the window of the envelope is 2.875 inches less than its length x. Create the function $A(x)$ that represents the area of the window in square inches. Simplify, if possible.
b. Evaluate $A(5.25)$ and explain what this value represents.
c. Evaluate $A(10)$. Is this possible for this particular envelope? Explain.

Refer to the table below for Exercises 115 and 116. It illustrates the average federal funds rate for the month of January (2009 to 2017).

YEAR	FED. RATE
2009	5.45
2010	5.98
2011	1.73
2012	1.24
2013	1.00
2014	2.25
2015	4.50
2016	5.25
2017	3.50

115. Finance. Is the relation whose domain is the year and whose range is the average federal funds rate for the month of January a function? Explain.

116. Finance. Write five ordered pairs whose domain is the set of even years from 2009–2017 and whose range is the set of corresponding average federal funds rate for the month of January.

FEATURE	BENEFIT TO STUDENT
Chapter-Opening Vignette	Piques the student's interest with a real-world application of material presented in the chapter. Later in the chapter, the concept from the vignette is reinforced.
Chapter Overview, Flowchart, and Learning Objectives	Allows students to see the big picture of how topics relate, and overarching learning objectives are presented.
Skills and Conceptual Objectives	Skills objectives represent what students should be able to do. Conceptual objectives emphasize a higher-level, global perspective of concepts.
Clear, Concise, and Inviting Writing Style, Tone, and Layout	Enables students to understand what they are reading, which reduces math anxiety and promotes student success.
Parallel Words and Math	Increases students' ability to read and understand examples with a seamless representation of their instructor's class (instructor's voice and what they would write on the board).
Common Mistakes	Addresses a different learning style: teaching by counterexample. Demonstrates common mistakes so that students understand why a step is incorrect and reinforces the correct mathematics.
Color for Pedagogical Reasons	Particularly helpful for visual learners when they see a function written in red and then its corresponding graph in red or a function written in blue and then its corresponding graph in blue.
Study Tips	Reinforces specific notes that you would want to emphasize in class.
Author Videos	Gives students a mini class of several examples worked by the author.
Your Turn	Engages students during class, builds student confidence, and assists instructor in real-time assessment.
Concept Checks	Reinforces concept learning objectives, much as Your Turn features reinforce skill learning objectives.
Catch the Mistake Exercises	Encourages students to assume the role of teacher—demonstrating a higher mastery level.
Conceptual Exercises	Teaches students to think more globally about a topic.
Inquiry-Based Learning Project (online only)	Lets students *discover* a mathematical identity, formula, and the like that is derived in the book.
Modeling Our World (online only)	Engages students in a modeling project of a timely subject: global climate change.
Chapter Review	Presents key ideas and formulas section by section in a chart. Improves study skills.
Chapter Review Exercises	Improves study skills.
Chapter Practice Test	Offers self-assessment and improves study skills.
Cumulative Test	Improves retention.

INSTRUCTOR SUPPLEMENTS

INSTRUCTOR'S SOLUTIONS MANUAL (ISBN: 978-1-119-27343-1)

- Contains worked-out solutions to all exercises in the text.

INSTRUCTOR'S MANUAL

Authored by Cynthia Young, the manual provides practical advice on teaching with the text, including:

- sample lesson plans and homework assignments
- suggestions for the effective utilization of additional resources and supplements
- sample syllabi
- Cynthia Young's Top 10 Teaching Tips & Tricks
- online component featuring the author presenting these Tips & Tricks

ANNOTATED INSTRUCTOR'S EDITION (ISBN: 978-1-119-27346-2)

- Displays answers to the vast majority of exercise questions in the back of the book.
- Provides additional classroom examples within the standard difficulty range of the in-text exercises, as well as challenge problems to assess your students' mastery of the material.

POWERPOINT SLIDES

- For each section of the book, a corresponding set of lecture notes and worked- out examples are presented as PowerPoint slides, available on the Book Companion Site (www.wiley.com/college/young) and *WileyPLUS*.

TEST BANK

- Contains approximately 900 questions and answers from every section of the text.

COMPUTERIZED TEST BANK

Electonically enhanced version of the Test Bank that

- contains approximately 900 algorithmically generated questions.
- allows instructors to freely edit, randomize, and create questions.
- allows instructors to create and print different versions of a quiz or exam.
- recognizes symbolic notation.

BOOK COMPANION WEBSITE (WWW.WILEY.COM/COLLEGE/YOUNG)

- Contains all instructor supplements listed plus a selection of personal response system questions.

WILEYPLUS

- *WileyPLUS* online homework features a full-service, digital learning environment, including additional resources for students, such as lecture videos by the author, self-practice exercises, tutorials, integrated links between the online text and supplements, and new interactive animations and ORION adaptive practice.
- *WileyPLUS* has been substantially revised and improved since the third edition of *Algebra and Trigonometry*. It now includes ORION, an adaptive practice engine built directly into *WileyPLUS* that can connect directly into the *WileyPLUS* gradebook, or into your campus Learning Management System gradebook if you select that option. Wiley has been incorporating ORION into *WileyPLUS* courses for over five years, including the Young *Precalculus* program. ORION brings the power of adaptive learning, which will continue to help students and instructors "bridge the gap."

STUDENT SUPPLEMENTS

STUDENT SOLUTIONS MANUAL (ISBN: 978-1-119-27342-4)

- Includes worked-out solutions for all odd problems in the text.

BOOK COMPANION WEBSITE (WWW.WILEY.COM/COLLEGE/YOUNG)

- Provides additional resources for students to enhance the learning experience.

ACKNOWLEDGMENTS

I want to express my sincerest gratitude to the entire Wiley team. I've said this before, and I will say it again: Wiley is the right partner for me. There is a reason that my dog is named Wiley—she's smart, competitive, a team player, and most of all, a joy to be around. There are several people within Wiley to whom I feel the need to express my appreciation: first and foremost to Laurie Rosatone, who convinced Wiley Higher Ed to invest in a young assistant professor's vision for a series and who has been unwavering in her commitment to student learning. To my editor Joanna Dingle, whose judgment I trust in both editorial and preschool decisions; thank you for surpassing my greatest expectations for an editor. To the rest of the math editorial team (Jennifer Lartz, Anne Scanlan-Rohrer, and Ryann Dannelly), you are all first class! This revision was planned and executed exceptionally well thanks to you. To the math marketing manager John LaVacca, thank you for helping reps tell my story: you are outstanding at your job. To product designer David Dietz, many thanks for your role in developing the online course and digital assets. To Mary Sanger, thank you for your attention to detail. To Maureen Eide, thank you for the new design! And finally, I'd like to thank all of the Wiley reps: thank you for your commitment to my series and your tremendous efforts to get professors to adopt this book for their students.

I would also like to thank all of the contributors who helped us make this *an even better book*. I'd first like to thank Mark McKibben. He is known as the author of the solutions manuals that accompany this series, but he is much more than that. Mark, thank you for making this series a priority, for being so responsive, and most of all for being my "go-to" person to think through ideas. I'd also like to thank Jodi B.A. McKibben, who is a statistician and teamed with Mark to develop the regression material. I'd like to thank Steve Davis, who was the inspiration for the Inquiry-Based Learning Projects (IBLPs) and a huge thanks to Lyn Riverstone, who developed all of the IBLPs. Special thanks to Laura Watkins for finding applications that are real and timely and to Ricki Alexander for updating all of the Technology Tips. I'd also like to thank Becky Schantz for her environmental problems (I now use AusPens because of Becky). Many thanks to Marie Vanisko, Jennifer Blue, and Diane Cook for accuracy checking of the text, exercises, and solutions.

I'd also like to thank the following reviewers, whose input helped make this book even better.

Text Reviewers

SANDY CARLSON, Pennsylvania College of Technology
ZE-LI DOU, Texas Christian University
DEBORAH LYNN DOUCETTE, Erie Community College – North Campus
SCOTT GENTILE, Hunter College, CUNY
MIKE KIRBY, Tidewater Community College
BRIDGETT LEE, Georgia Southern University
LEBBIE LEE LIGON, Piedmont Technical College
KELLY PEARSON, Murray State University
COLLEEN QUINN, Erie Community College
RACHEL RADER, Ohio Northern University
SEAN SMITH, California State Polytechnic University, Pomona

Animations Reviewers

MONIKA H. CHAMPION, Ivy Tech Community College
KIM CHRISTENSEN, Metropolitan Community College
YAMIR DEJESUS-DECENA, Dutchess Community College
MARISSA FORD, Ivy Tech Community College
BARBARA HESS, California University of Pennsylvania
CHRISTINA K. HOUSTON, Community College of Allegheny County
PHYLLIS LEFTON, Manhattanville College
CARRIE MCCAMMON, Ivy Tech Community College
HOLLY J. MIDDLETON, University of North Florida
BECKY MOENING, Ivy Tech Community College, Warsaw
DENISE RACE, Eastfield College
EDWARD SCHWARTZ, Manhattanville College
MIKE SHIRAZI, Germanna Community College
MISTY VORDER, Hillsborough Community College, Brandon

Contents

fotografixx/Getty Images

[0] Prerequisites and Review 2

0.1 REAL NUMBERS 4
0.2 INTEGER EXPONENTS AND SCIENTIFIC NOTATION 17
0.3 POLYNOMIALS: BASIC OPERATIONS 27
0.4 FACTORING POLYNOMIALS 36
0.5 RATIONAL EXPRESSIONS 46
0.6 RATIONAL EXPONENTS AND RADICALS 60
0.7 COMPLEX NUMBERS 70

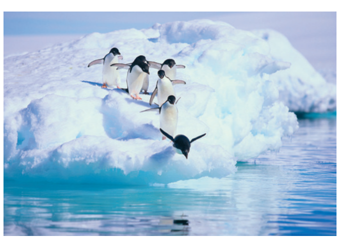
Review 77 | Review Exercises 79 | Practice Test 81

Hero Images/Getty Images, Inc.

[1] Equations and Inequalities 82

1.1 LINEAR EQUATIONS 84
1.2 APPLICATIONS INVOLVING LINEAR EQUATIONS 93
1.3 QUADRATIC EQUATIONS 107
1.4 OTHER TYPES OF EQUATIONS 121
1.5 LINEAR INEQUALITIES 129
1.6 POLYNOMIAL AND RATIONAL INEQUALITIES 139
1.7 ABSOLUTE VALUE EQUATIONS AND INEQUALITIES 149

Review 157 | Review Exercises 158 | Practice Test 162 | Cumulative Test 163



Fuse/Getty Images

[2] Graphs 164

2.1 BASIC TOOLS: CARTESIAN PLANE, DISTANCE, AND MIDPOINT 166
2.2 GRAPHING EQUATIONS, POINT-PLOTTING, INTERCEPTS, AND SYMMETRY 173
2.3 LINES 186
2.4 CIRCLES 202
2.5* LINEAR REGRESSION: BEST FIT 209

Review 231 | Review Exercises 232 | Practice Test 234 | Cumulative Test 235

SuperStock/Alamy Stock Photo

[3] Functions and Their Graphs 236

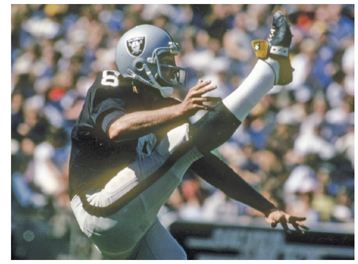
3.1 FUNCTIONS 238
3.2 GRAPHS OF FUNCTIONS; PIECEWISE-DEFINED FUNCTIONS; INCREASING AND DECREASING FUNCTIONS; AVERAGE RATE OF CHANGE 255
3.3 GRAPHING TECHNIQUES: TRANSFORMATIONS 274
3.4 OPERATIONS ON FUNCTIONS AND COMPOSITION OF FUNCTIONS 287
3.5 ONE-TO-ONE FUNCTIONS AND INVERSE FUNCTIONS 298
3.6 MODELING FUNCTIONS USING VARIATION 312

Review 322 | Review Exercises 324 | Practice Test 328 | Cumulative Test 329

[4] Polynomial and Rational Functions 330

4.1 QUADRATIC FUNCTIONS 332
4.2 POLYNOMIAL FUNCTIONS OF HIGHER DEGREE 349
4.3 DIVIDING POLYNOMIALS: LONG DIVISION AND SYNTHETIC DIVISION 363
4.4 THE REAL ZEROS OF A POLYNOMIAL FUNCTION 372
4.5 COMPLEX ZEROS: THE FUNDAMENTAL THEOREM OF ALGEBRA 388
4.6 RATIONAL FUNCTIONS 396

Review 416 | Review Exercises 418 | Practice Test 422 | Cumulative Test 423



Focus On Sport/Getty Images, Inc.

[5] Exponential and Logarithmic Functions 424

5.1 EXPONENTIAL FUNCTIONS AND THEIR GRAPHS 426
5.2 LOGARITHMIC FUNCTIONS AND THEIR GRAPHS 440
5.3 PROPERTIES OF LOGARITHMS 456
5.4 EXPONENTIAL AND LOGARITHMIC EQUATIONS 464
5.5 EXPONENTIAL AND LOGARITHMIC MODELS 474

Review 485 | Review Exercises 487 | Practice Test 490 | Cumulative Test 491

Francois Gohier/Science Source

[6] Trigonometric Functions 492

6.1 ANGLES, DEGREES, AND TRIANGLES 494
6.2 DEFINITION 1 OF TRIGONOMETRIC FUNCTIONS: RIGHT TRIANGLE RATIOS 511
6.3 APPLICATIONS OF RIGHT TRIANGLE TRIGONOMETRY: SOLVING RIGHT TRIANGLES 525
6.4 DEFINITION 2 OF TRIGONOMETRIC FUNCTIONS: CARTESIAN PLANE 538
6.5 TRIGONOMETRIC FUNCTIONS OF NONACUTE ANGLES 552
6.6 RADIAN MEASURE AND APPLICATIONS 567
6.7 DEFINITION 3 OF TRIGONOMETRIC FUNCTIONS: UNIT CIRCLE APPROACH 584
6.8 GRAPHS OF SINE AND COSINE FUNCTIONS 594
6.9 GRAPHS OF OTHER TRIGONOMETRIC FUNCTIONS 623

Review 641 | Review Exercises 647 | Practice Test 650 | Cumulative Test 651

Richard Cummins / Getty Images, Inc.

[7] Analytic Trigonometry 652

7.1 BASIC TRIGONOMETRIC IDENTITIES 654
7.2 VERIFYING TRIGONOMETRIC IDENTITIES 665
7.3 SUM AND DIFFERENCE IDENTITIES 675
7.4 DOUBLE-ANGLE IDENTITIES 689
7.5 HALF-ANGLE IDENTITIES 697
7.6 PRODUCT-TO-SUM AND SUM-TO-PRODUCT IDENTITIES 708
7.7 INVERSE TRIGONOMETRIC FUNCTIONS 716
7.8 TRIGONOMETRIC EQUATIONS 737

Review 754 | Review Exercises 757 | Practice Test 762 | Cumulative Test 763

laurentiu iordache / Alamy Stock Photo

[8] Additional Topics in Trigonometry 764

8.1 OBLIQUE TRIANGLES AND THE LAW OF SINES 766
8.2 THE LAW OF COSINES 780
8.3 THE AREA OF A TRIANGLE 791
8.4 VECTORS 798
8.5 THE DOT PRODUCT 813
8.6 POLAR (TRIGONOMETRIC) FORM OF COMPLEX NUMBERS 821
8.7 PRODUCTS, QUOTIENTS, POWERS, AND ROOTS OF COMPLEX NUMBERS; DE MOIVRE'S THEOREM 829
8.8 POLAR EQUATIONS AND GRAPHS 841

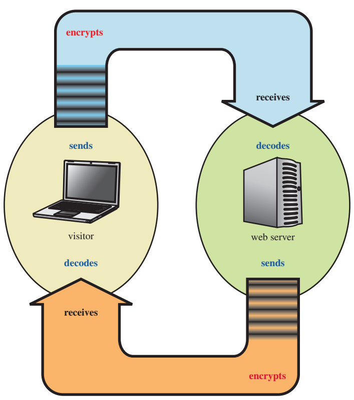
Review 856 | Review Exercises 859 | Practice Test 862 | Cumulative Test 863

PAUL J. RICHARDS/
Getty Images, Inc.

[9] Systems of Linear Equations and Inequalities 864

9.1 SYSTEMS OF LINEAR EQUATIONS IN TWO VARIABLES 866
9.2 SYSTEMS OF LINEAR EQUATIONS IN THREE VARIABLES 881
9.3 PARTIAL FRACTIONS 892
9.4 SYSTEMS OF LINEAR INEQUALITIES IN TWO VARIABLES 903
9.5 THE LINEAR PROGRAMMING MODEL 914

Review 921 | Review Exercises 922 | Practice Test 924 | Cumulative Test 925



[10] Matrices 926

10.1 MATRICES AND SYSTEMS OF LINEAR EQUATIONS 928
10.2 MATRIX ALGEBRA 950
10.3 MATRIX EQUATIONS; THE INVERSE OF A SQUARE MATRIX 964
10.4 THE DETERMINANT OF A SQUARE MATRIX AND CRAMER'S RULE 976

Review 990 | Review Exercises 993 | Practice Test 996 | Cumulative Test 997

[11] Analytic Geometry and Systems of Nonlinear Equations and Inequalities 998

11.1 CONIC BASICS 1000
11.2 THE PARABOLA 1003
11.3 THE ELLIPSE 1015
11.4 THE HYPERBOLA 1028
11.5 SYSTEMS OF NONLINEAR EQUATIONS 1039
11.6 SYSTEMS OF NONLINEAR INEQUALITIES 1049
11.7 ROTATION OF AXES 1057
11.8 POLAR EQUATIONS OF CONICS 1067
11.9 PARAMETRIC EQUATIONS AND GRAPHS 1078

Review 1086 | Review Exercises 1089 | Practice Test 1093 | Cumulative Test 1095

Ribeiro antonio/Shutterstock

[12] Sequences, Series, and Probability 1096

12.1 SEQUENCES AND SERIES 1098
12.2 ARITHMETIC SEQUENCES AND SERIES 1109
12.3 GEOMETRIC SEQUENCES AND SERIES 1117
12.4 MATHEMATICAL INDUCTION 1128
12.5 THE BINOMIAL THEOREM 1133
12.6 COUNTING, PERMUTATIONS, AND COMBINATIONS 1141
12.7 PROBABILITY 1151

Review 1160 | Review Exercises 1162 | Practice Test 1166 | Cumulative Test 1167

A BASIC STRATEGY FOR BLACKJACK

Dealer's Up Card

Your Hand		2	3	4	5	6	7	8	9	10	A
I	17+	S	S	S	S	S	S	S	S	S	S
	16	S	S	S	S	S	H	H	H	H	H
	15	S	S	S	S	S	H	H	H	H	H
	14	S	S	S	S	S	H	H	H	H	H
	13	S	S	S	S	S	H	H	H	H	H
	12	H	H	S	S	S	H	H	H	H	H
II	11	D	D	D	D	D	D	D	D	D	H
	10	D	D	D	D	D	D	D	D	H	H
	9	H	D	D	D	D	H	H	H	H	H
	5 - 8	H	H	H	H	H	H	H	H	H	H
III	A, 8 - 10	S	S	S	S	S	S	S	S	S	S
	A, 7	S	D	D	D	D	S	S	H	H	H
	A, 6	H	D	D	D	D	H	H	H	H	H
	A, 5	H	H	D	D	D	H	H	H	H	H
	A, 4	H	H	D	D	D	H	H	H	H	H
	A, 3	H	H	H	D	D	H	H	H	H	H
	A, 2	H	H	H	D	D	H	H	H	H	H
IV	A, A; 8, 8	SP	SP	SP	SP	SP	SP	SP	SP	SP	SP
	10, 10	S	S	S	S	S	S	S	S	S	S
	9, 9	SP	SP	SP	SP	SP	S	SP	SP	S	S
	7, 7	SP	SP	SP	SP	SP	SP	H	H	H	H
	6, 6	H	SP	SP	SP	SP	H	H	H	H	H
	5, 5	D	D	D	D	D	D	D	D	H	H
	4, 4	H	H	H	H	H	H	H	H	H	H
	3, 3	H	H	SP	SP	SP	SP	H	H	H	H
	2, 2	H	H	SP	SP	SP	SP	H	H	H	H

When surrender is allowed, surrender 9, 7 or 10, 6 vs 9, 10, A; 9, 6 or 10, 5 vs 10
When doubling down after splitting is allowed, split: 2's, 3's, 7's vs 2-7; 4's vs 5 or 6; 6's vs 2-6

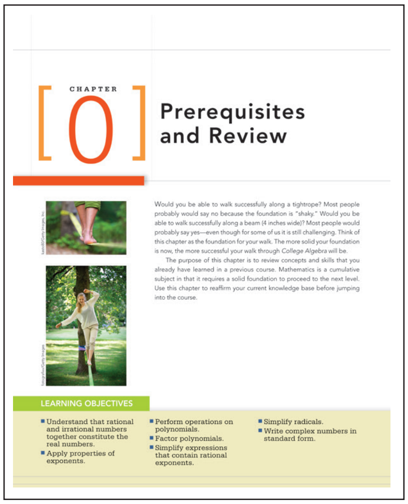
HIT STAND DOUBLE DOWN SPLIT

Answers to Odd-Numbered Exercises 1169
Applications Index 1261
Subject Index 1265

A Note from the Author

TO THE STUDENT

I wrote this text with careful attention to ways in which to make your learning experience more successful. If you take full advantage of the unique features and elements of this textbook, I believe your experience will be fulfilling and enjoyable. Let's walk through some of the special book features that will help you in your study of College Algebra.

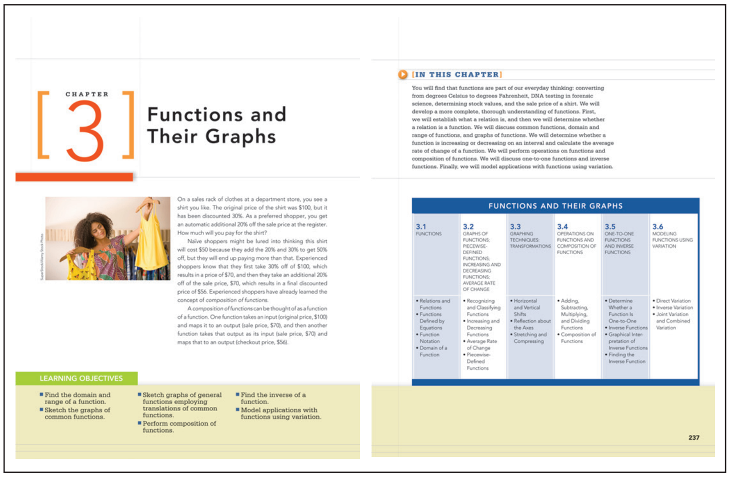


Prerequisites and Review [Chapter 0]

A comprehensive review of prerequisite knowledge (intermediate algebra topics) in Chapter 0 provides a brushup on knowledge and skills necessary for success in the course.

Clear, Concise, and Inviting Writing

Special attention has been paid to presenting an engaging, clear, precise narrative in a layout that is easy to use and designed to reduce any math anxiety you may have.



Chapter Introduction, Flowchart, Section Headings, and Objectives

An opening vignette, flowchart, list of chapter sections, and chapter learning objectives give you an overview of the chapter.

3.3 GRAPHING TECHNIQUES: TRANSFORMATIONS

SKILLS OBJECTIVES	CONCEPTUAL OBJECTIVES
■ Sketch the graph of a function using horizontal and vertical shifting of common functions. ■ Sketch the graph of a function by reflecting a common function about the x-axis or y-axis. ■ Sketch the graph of a function by stretching or compressing a common function.	■ Understand why a shift in the argument inside the function corresponds to a horizontal shift and a shift outside the function corresponds to a vertical shift. ■ Understand why a negative argument inside the function corresponds to a reflection about the y-axis and a negative outside the function corresponds to a reflection about the x-axis. ■ Understand the difference between rigid and nonrigid transformations.

Skills and Conceptual Objectives

For every section, objectives are further divided by skills *and* concepts so you can see the difference between solving problems and truly understanding concepts.

Examples

Examples pose a specific problem using concepts already presented and then work through the solution. These serve to enhance your understanding of the subject matter.

Your Turn

Immediately following many examples, you are given a similar problem to reinforce and check your understanding. This helps build confidence as you progress in the chapter. These are ideal for in-class activity or for preparing for homework later. Answers are provided in the margin for a quick check of your work.

EXAMPLE 9 Evaluating the Difference Quotient

For the function $f(x) = x^2 - x$, find $\frac{f(x+h) - f(x)}{h}$, $h \neq 0$.

Solution:

Use placeholder notation for the function $f(x) = x^2 - x$. $f(\square) = (\square)^2 - (\square)$

Calculate $f(x+h)$. $f(x+h) = (x+h)^2 - (x+h)$

Write the difference quotient. $\frac{f(x+h) - f(x)}{h}$

Let $f(x+h) = (x+h)^2 - (x+h)$ and $f(x) = x^2 - x$.

$$\frac{f(x+h) - f(x)}{h} = \frac{\overbrace{[(x+h)^2 - (x+h)]}^{f(x+h)} - \overbrace{[x^2 - x]}^{f(x)}}{h} \quad h \neq 0$$

Eliminate the parentheses inside the first set of brackets. $= \frac{[x^2 + 2xh + h^2 - x - h] - [x^2 - x]}{h}$

Eliminate the brackets in the numerator. $= \frac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h}$

Combine like terms. $= \frac{2xh + h^2 - h}{h}$

Factor the numerator. $= \frac{h(2x + h - 1)}{h}$

Divide out the common factor, h. $= \boxed{2x + h - 1} \quad h \neq 0$

YOUR TURN Evaluate the difference quotient for $f(x) = x^2 - 1$.

Common Mistake/Correct Versus Incorrect

In addition to standard examples, some problems are worked out both correctly and incorrectly to highlight common errors. Counterexamples like these are often an effective learning approach.

common mistake

A common misunderstanding is to interpret the notation $f(x+1)$ as a sum: $f(x+1) \neq f(x) + f(1)$.

✓CORRECT

Write the original function.

$f(x) = x^2 - 3x$

Replace the argument x with a placeholder.

$f(\square) = (\square)^2 - 3(\square)$

Substitute $x + 1$ for the argument.

$f(x+1) = (x+1)^2 - 3(x+1)$

Eliminate the parentheses.

$f(x+1) = x^2 + 2x + 1 - 3x - 3$

Combine like terms.

$\boxed{f(x+1) = x^2 - x - 2}$

✖INCORRECT

The **ERROR** is in interpreting the notation as a sum.

$f(x+1) \neq f(x) + f(1)$

$\neq x^2 - 3x - 2$

Parallel Words and Math

This text reverses the common textbook presentation of examples by placing the explanation in words *on the left* and the mathematics in parallel *on the right*. This makes it easier to read through examples as the material flows more naturally from left to right and as commonly presented in class.

WORDS	MATH
Let the difference between x_1 and x_2 be h.	$x_2 - x_1 = h$
Solve for x_2.	$x_2 = x_1 + h$
Substitute $x_2 - x_1 = h$ into the denominator and	Average rate of change $= \frac{f(x_2) - f(x_1)}{x_2 - x_1}$
$x_2 = x_1 + h$ into the numerator of the average rate of change.	$= \frac{f(x_1 + h) - f(x_1)}{h}$
Let $x_1 = x$.	$= \boxed{\frac{f(x+h) - f(x)}{h}}$

Study Tips and Caution Notes

These marginal reminders call out important hints or warnings to be aware of related to the topic or problem.

STUDY TIP

Use brackets or parentheses around $f(x)$ to avoid forgetting to distribute the negative sign:

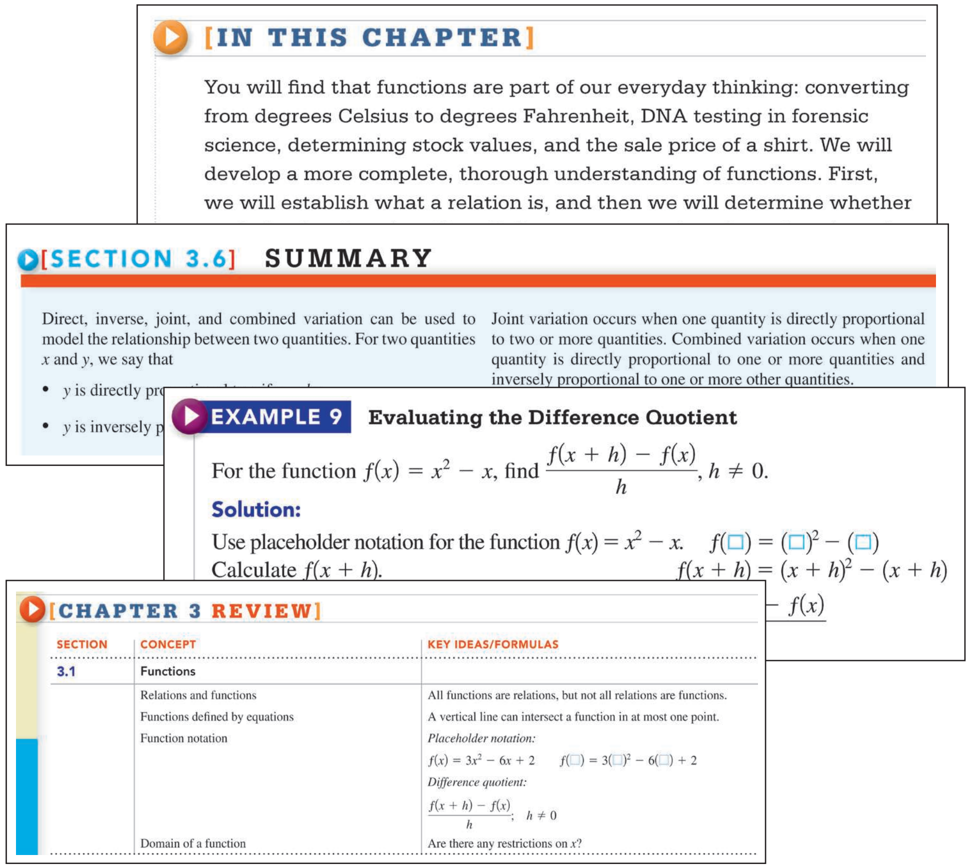
$$\frac{f(x+h) - [f(x)]}{h}$$

CAUTION

$f \circ g \neq f \cdot g$

Video Icons

Video icons appear on all chapter introductions, chapter and section reviews, as well as selected examples throughout the chapter to indicate that the author has created a video segment for that element. These video clips help you work through the selected examples with the author as your "private tutor."



[IN THIS CHAPTER]

You will find that functions are part of our everyday thinking: converting from degrees Celsius to degrees Fahrenheit, DNA testing in forensic science, determining stock values, and the sale price of a shirt. We will develop a more complete, thorough understanding of functions. First, we will establish what a relation is, and then we will determine whether

[SECTION 3.6] SUMMARY

Direct, inverse, joint, and combined variation can be used to model the relationship between two quantities. For two quantities x and y, we say that

Joint variation occurs when one quantity is directly proportional to two or more quantities. Combined variation occurs when one quantity is directly proportional to one or more quantities and inversely proportional to one or more other quantities.

EXAMPLE 9 Evaluating the Difference Quotient

For the function $f(x) = x^2 - x$, find $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$.

Solution:

Use placeholder notation for the function $f(x) = x^2 - x$. $f(\square) = (\square)^2 - (\square)$

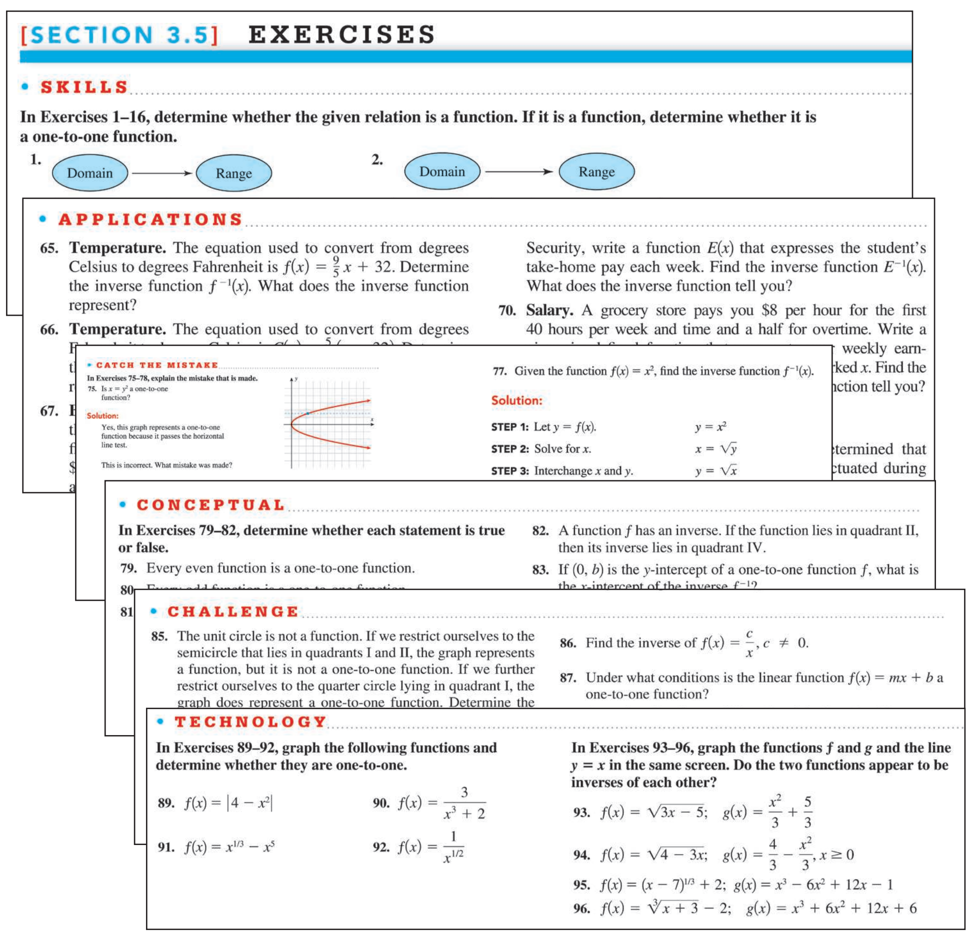
Calculate $f(x + h)$. $f(x+h) = (x+h)^2 - (x+h)$

[CHAPTER 3 REVIEW]

SECTION	CONCEPT	KEY IDEAS/FORMULAS
3.1	Functions	
	Relations and functions	All functions are relations, but not all relations are functions.
	Functions defined by equations	A vertical line can intersect a function in at most one point.
	Function notation	*Placeholder notation:* $f(x) = 3x^2 - 6x + 2$ $f(\square) = 3(\square)^2 - 6(\square) + 2$ *Difference quotient:* $\dfrac{f(x+h) - f(x)}{h}$; $h \neq 0$
	Domain of a function	Are there any restrictions on x?

Six Types of Exercises

Every text section ends with **Skills, Applications, Catch the Mistake, Conceptual, Challenge, and Technology** exercises. The exercises gradually increase in difficulty and vary in skill and conceptual emphasis. Catch the Mistake exercises increase the depth of understanding and reinforce what you have learned. Conceptual and Challenge exercises specifically focus on assessing conceptual understanding. Technology exercises enhance your understanding and ability using scientific and graphing calculators.



[SECTION 3.5] EXERCISES

• SKILLS

In Exercises 1–16, determine whether the given relation is a function. If it is a function, determine whether it is a one-to-one function.

1. Domain → Range 2. Domain → Range

• APPLICATIONS

65. **Temperature.** The equation used to convert from degrees Celsius to degrees Fahrenheit is $f(x) = \frac{9}{5}x + 32$. Determine the inverse function $f^{-1}(x)$. What does the inverse function represent?

66. **Temperature.** The equation used to convert from degrees

Security, write a function $E(x)$ that expresses the student's take-home pay each week. Find the inverse function $E^{-1}(x)$. What does the inverse function tell you?

70. **Salary.** A grocery store pays you $8 per hour for the first 40 hours per week and time and a half for overtime. Write a

• CATCH THE MISTAKE

In Exercises 75–78, explain the mistake that is made.

75. Is $x = y^2$ a one-to-one function?

Solution:

Yes, this graph represents a one-to-one function because it passes the horizontal line test.

This is incorrect. What mistake was made?

77. Given the function $f(x) = x^2$, find the inverse function $f^{-1}(x)$.

Solution:

STEP 1: Let $y = f(x)$. $y = x^2$

STEP 2: Solve for x. $x = \sqrt{y}$

STEP 3: Interchange x and y. $y = \sqrt{x}$

• CONCEPTUAL

In Exercises 79–82, determine whether each statement is true or false.

79. Every even function is a one-to-one function.

82. A function f has an inverse. If the function lies in quadrant II, then its inverse lies in quadrant IV.

83. If $(0, b)$ is the y-intercept of a one-to-one function f, what is

• CHALLENGE

85. The unit circle is not a function. If we restrict ourselves to the semicircle that lies in quadrants I and II, the graph represents a function, but it is not a one-to-one function. If we further restrict ourselves to the quarter circle lying in quadrant I, the graph does represent a one-to-one function. Determine the

86. Find the inverse of $f(x) = \frac{c}{x}$, $c \neq 0$.

87. Under what conditions is the linear function $f(x) = mx + b$ a one-to-one function?

• TECHNOLOGY

In Exercises 89–92, graph the following functions and determine whether they are one-to-one.

89. $f(x) = |4 - x^2|$

90. $f(x) = \dfrac{3}{x^3 + 2}$

91. $f(x) = x^{1/3} - x^5$

92. $f(x) = \dfrac{1}{x^{1/2}}$

In Exercises 93–96, graph the functions f and g and the line $y = x$ in the same screen. Do the two functions appear to be inverses of each other?

93. $f(x) = \sqrt{3x - 5}$; $g(x) = \dfrac{x^2}{3} + \dfrac{5}{3}$

94. $f(x) = \sqrt{4 - 3x}$; $g(x) = \dfrac{4}{3} - \dfrac{x^2}{3}$, $x \geq 0$

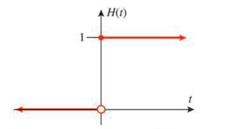
95. $f(x) = (x - 7)^{1/3} + 2$; $g(x) = x^3 - 6x^2 + 12x - 1$

96. $f(x) = \sqrt[3]{x+3} - 2$; $g(x) = x^3 + 6x^2 + 12x + 6$

[CONCEPT CHECK]

If the domain consists of all physical (home) addresses in a particular county and the range is the persons living in that county, does this describe a relation? And if so, is that relation a function?

ANSWER This is a relation but not a function.



[CONCEPT CHECK]

State the domain, range, and any points of discontinuity for the Heaviside function.

ANSWER Domain: $(-\infty, \infty)$
Range: $[0] \cup [1]$
Point of Discontinuity: $x = 0$

[CONCEPT CHECK]

Given a fixed distance, the time it takes you to drive that distance varies inversely with ________?

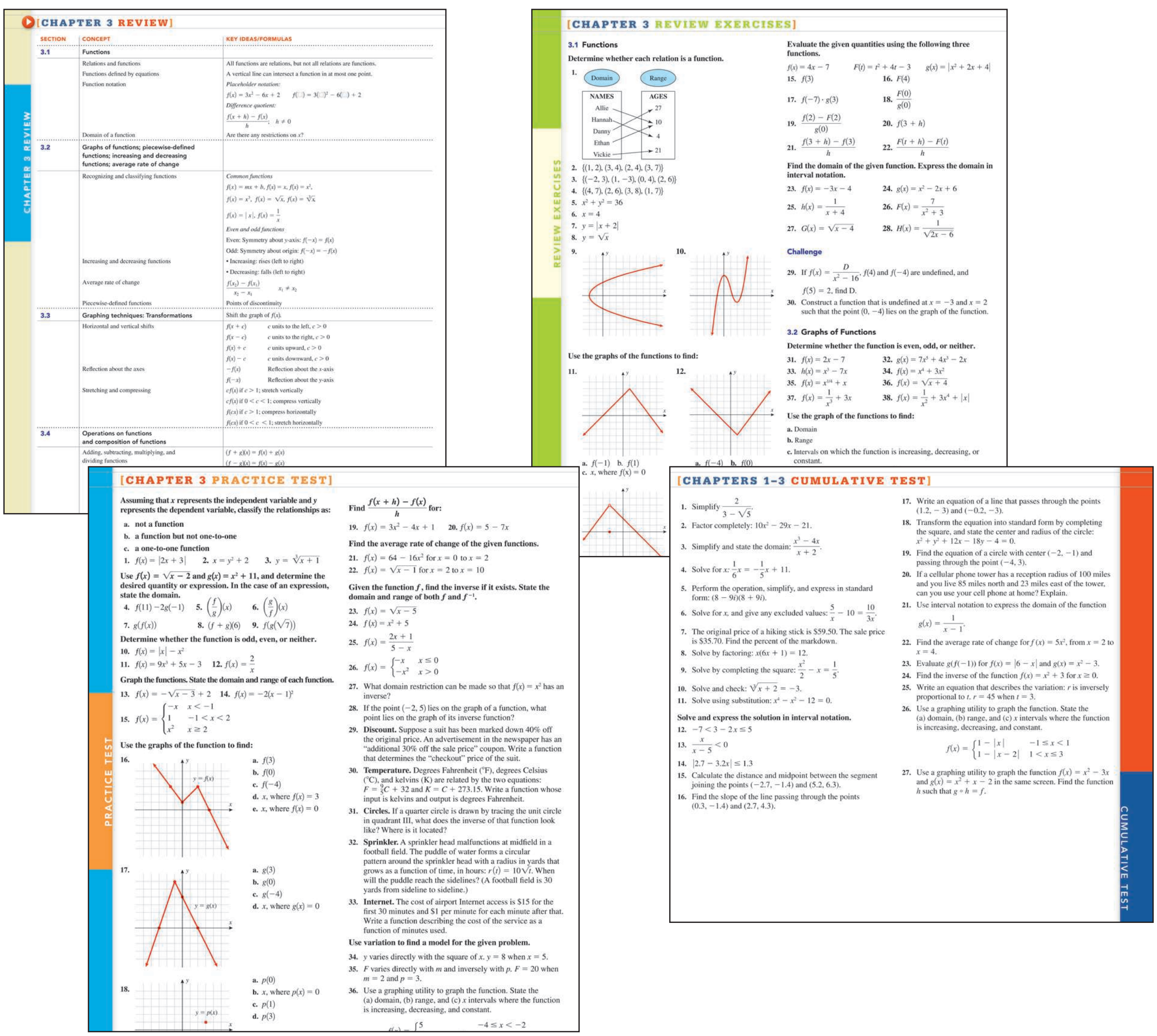
ANSWER rate (or speed)

Concept Check Questions

Similar to how Your Turn features reinforce skill learning objectives, Concept Checks reinforce concept learning objectives.

Chapter Review, Review Exercises, Practice Test, Cumulative Test

At the end of every chapter, a summary review chart organizes the key learning concepts in an easy-to-use one- or two-page layout. This feature includes key ideas and formulas, as well as indicating relevant pages and review exercises so that you can quickly summarize a chapter and study smarter. Review Exercises, arranged by section heading, are provided for extra study and practice. A Practice Test, without section headings, offers even more self-practice before moving on. A new Cumulative Test feature offers study questions based on all previous chapters' content, thus helping you build upon previously learned concepts.



CHAPTER

0 Prerequisites and Review

kasto80/Getty Images, Inc.

fotografixx/Getty Images

Would you be able to walk successfully along a tightrope? Most people probably would say no because the foundation is "shaky." Would you be able to walk successfully along a beam (4 inches wide)? Most people would probably say yes—even though for some of us it is still challenging. Think of this chapter as the foundation for your walk. The more solid your foundation is now, the more successful your walk through *College Algebra* will be.

The purpose of this chapter is to review concepts and skills that you already have learned in a previous course. Mathematics is a cumulative subject in that it requires a solid foundation to proceed to the next level. Use this chapter to reaffirm your current knowledge base before jumping into the course.

LEARNING OBJECTIVES

- Understand that rational and irrational numbers together constitute the real numbers.
- Apply properties of exponents.
- Perform operations on polynomials.
- Factor polynomials.
- Simplify expressions that contain rational exponents.
- Simplify radicals.
- Write complex numbers in standard form.

[IN THIS CHAPTER]

Real numbers, integer exponents, and scientific notation will be discussed, followed by rational exponents and radicals. Simplification of radicals and rationalization of denominators will be reviewed. Basic operations such as addition, subtraction, and multiplication of polynomials will be discussed followed by a review of how to factor polynomials. Rational expressions will be discussed, and a brief overview of solving simple algebraic equations will be given. After reviewing all of these aspects of real numbers, this chapter will conclude with a review of complex numbers.

PREREQUISITES AND REVIEW

0.1 REAL NUMBERS	0.2 INTEGER EXPONENTS AND SCIENTIFIC NOTATION	0.3 POLYNOMIALS: BASIC OPERATIONS	0.4 FACTORING POLYNOMIALS	0.5 RATIONAL EXPRESSIONS	0.6 RATIONAL EXPONENTS AND RADICALS	0.7 COMPLEX NUMBERS
• The Set of Real Numbers • Approximations: Rounding and Truncation • Order of Operations • Properties of Real Numbers	• Integer Exponents • Scientific Notation	• Adding and Subtracting Polynomials • Multiplying Polynomials • Special Products	• Greatest Common Factor • Factoring Formulas: Special Polynomial Forms • Factoring a Trinomial as a Product of Two Binomials • Factoring by Grouping • A Strategy for Factoring Polynomials	• Rational Expressions and Domain Restrictions • Simplifying Rational Expressions • Multiplying and Dividing Rational Expressions • Adding and Subtracting Rational Expressions • Complex Rational Expressions	• Square Roots • Other (*n*th) Roots • Rational Exponents	• The Imaginary Unit, *i* • Adding and Subtracting Complex Numbers • Multiplying Complex Numbers • Dividing Complex Numbers • Raising Complex Numbers to Integer Powers

0.1 REAL NUMBERS

SKILLS OBJECTIVES

- Classify real numbers as rational or irrational.
- Round or truncate real numbers.
- Simplify expressions and evaluate algebraic expressions using the correct order of operations.
- Apply properties of real numbers and basic rules of algebra in simplifying and evaluating expressions.

CONCEPTUAL OBJECTIVES

- Understand that rational and irrational numbers are mutually exclusive and complementary subsets of real numbers.
- Understand the difference between rounding and truncating decimal values and that the resulting approximations may or may not be equal.
- Learn the order of operations for real numbers.
- Know and understand the basic properties of real numbers and the basic rules of algebra.

0.1.1 The Set of Real Numbers

0.1.1 SKILL

Classify real numbers as rational or irrational.

0.1.1 CONCEPTUAL

Understand that rational and irrational numbers are mutually exclusive and complementary subsets of real numbers.

A **set** is a group or collection of objects that are called **members** or **elements** of the set. If *every* member of set B is also a member of set A, then we say B is a **subset** of A and denote it as $B \subset A$.

For example, the starting lineup on a baseball team is a subset of the entire team. The set of **natural numbers**, $\{1, 2, 3, 4, \ldots\}$, is a subset of the set of **whole numbers**, $\{0, 1, 2, 3, 4, \ldots\}$, which is a subset of the set of **integers**, $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, \ldots\}$, which is a subset of the set of *rational numbers*, which is a subset of the set of *real numbers*. The three dots, called an **ellipsis**, indicate that the pattern continues indefinitely.

If a set has no elements, it is called the **empty set**, or **null set**, and is denoted by the symbol $\varnothing$. The **set of real numbers** consists of two main subsets: *rational* and *irrational* numbers.

DEFINITION **Rational Number**

A **rational number** is a number that can be expressed as a quotient (ratio) of two integers, $\frac{a}{b}$, where the integer a is called the **numerator** and the integer b is called the **denominator** and where $b \neq 0$.

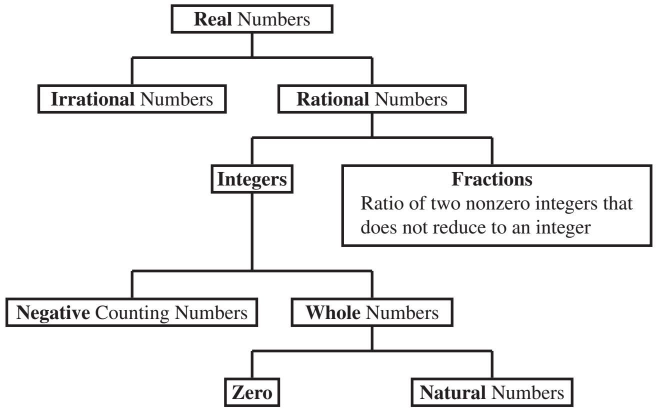
Rational numbers include all integers or all fractions that are ratios of integers. Note that any integer can be written as a ratio whose denominator is equal to 1. In decimal form, the rational numbers are those that terminate or are nonterminating with a repeated decimal pattern, which is represented with an overbar. Those decimals that do not repeat and do not terminate are **irrational numbers**. The numbers

$$5, \ -17, \ \frac{1}{3}, \ \sqrt{2}, \ \pi, \ 1.37, \ 0, \ -\frac{19}{17}, \ 3.66\overline{6}, \ 3.2179\ldots$$

are examples of **real** numbers, where $5, -17, \frac{1}{3}, 1.37, 0, -\frac{19}{17}$, and $3.66\overline{6}$ are rational numbers, and $\sqrt{2}, \pi$, and $3.2179\ldots$ are irrational numbers. It is important to note that the ellipsis following the last decimal digit denotes continuing in an irregular fashion, whereas the absence of such dots to the right of the last decimal digit implies that the decimal expansion terminates.

RATIONAL NUMBER (FRACTION)	CALCULATOR DISPLAY	DECIMAL REPRESENTATION	DESCRIPTION
$\frac{7}{2}$	3.5	3.5	Terminates
$\frac{15}{12}$	1.25	1.25	Terminates
$\frac{2}{3}$	0.666666666	$0.\overline{6}$	Repeats
$\frac{1}{11}$	0.09090909	$0.\overline{09}$	Repeats

Notice that the overbar covers the entire repeating pattern. The following figure and table illustrate the subset relationship and examples of different types of real numbers.

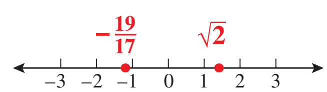


STUDY TIP

Every real number is either a rational number *or* an irrational number.

SYMBOL	NAME	DESCRIPTION	EXAMPLES
$\mathbb{N}$	Natural numbers	Counting numbers	$1, 2, 3, 4, 5, \ldots$
$\mathbb{W}$	Whole numbers	Natural numbers and zero	$0, 1, 2, 3, 4, 5, \ldots$
$\mathbb{Z}$	Integers	Whole numbers and negative natural numbers	$\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots$
$\mathbb{Q}$	Rational numbers	Ratios of integers: $\frac{a}{b} (b \neq 0)$ • Decimal representation terminates, or • Decimal representation repeats	$-17, -\frac{19}{7}, 0, \frac{1}{3}, 1.37, 3.6\overline{66}, 5$
$\mathbb{I}$	Irrational numbers	Numbers whose decimal representation does *not* terminate or repeat	$\sqrt{2}, 1.2179\ldots, \pi$
$\mathbb{R}$	Real numbers	Rational and irrational numbers	$\pi, 5, -\frac{2}{3}, 17.25, \sqrt{7}$

Since the set of real numbers can be formed by combining the set of rational numbers and the set of irrational numbers, then every real number is either rational or irrational. The set of rational numbers and the set of irrational numbers are mutually exclusive (no shared elements) and complementary sets. The **real number line** is a graph used to represent the set of all real numbers.



EXAMPLE 1 **Classifying Real Numbers**

Classify the following real numbers as rational or irrational:

$$-3, \quad 0, \quad \tfrac{1}{4}, \quad \sqrt{3}, \quad \pi, \quad 7.51, \quad \tfrac{1}{3}, \quad -\tfrac{8}{5}, \quad 6.666\overline{6}$$

Solution:

Rational: $-3, \quad 0, \quad \frac{1}{4}, \quad 7.51, \quad \frac{1}{3}, \quad -\frac{8}{5}, \quad -6.666\overline{6}$ Irrational: $\sqrt{3}, \pi$

YOUR TURN Classify the following real numbers as rational or irrational:

$$-\tfrac{7}{3}, 5.999\overline{9}, 12, 0, -5.27, \sqrt{5}, 2.010010001\ldots$$

[CONCEPT CHECK]

TRUE OR FALSE All integers are rational numbers.

ANSWER True

ANSWER

Rational: $-\frac{7}{3}, 5.999\overline{9}, 12, 0, -5.27$
Irrational: $\sqrt{5}, 2.010010001\ldots$

0.1.2 Approximations: Rounding and Truncation

0.1.2 SKILL

Round or truncate real numbers.

0.1.2 CONCEPTUAL

Understand the difference between rounding and truncating decimal values and know that the resulting approximations may or may not be equal.

Every real number can be represented by a decimal. When a real number is in decimal form, it can be approximated by either *rounding off* or *truncating* to a given decimal place. **Truncation** is "cutting off" or eliminating everything to the right of a certain decimal place. **Rounding** means looking to the right of the specified decimal place and making a judgment. If the digit to the right is greater than or equal to 5, then the specified digit is rounded up, or increased by one unit. If the digit to the right is less than 5, then the specified digit stays the same. In both of these cases all decimal places to the right of the specified place are removed.

[CONCEPT CHECK]

TRUE OR FALSE Truncating and rounding always have the same result.

ANSWER False

EXAMPLE 2 Approximating Decimals to Two Places

Approximate 17.368204 to two decimal places by

a. truncation **b.** rounding

Solution:

a. To truncate, eliminate all digits to the right of the 6. $\boxed{17.36}$

b. To round, look to the right of the 6.

Because "8" is greater than 5, round up (add 1 to the 6). $\boxed{17.37}$

YOUR TURN Approximate 23.02492 to two decimal places by

a. truncation **b.** rounding

ANSWER

a. Truncation: 23.02

b. Rounding: 23.02

EXAMPLE 3 Approximating Decimals to Four Places

Approximate 7.293516 to four decimal places by

a. truncation **b.** rounding

Solution:

The "5" is in the fourth decimal place.

a. To truncate, eliminate all digits to the right of 5. $\boxed{7.2935}$

b. To round, look to the right of the 5.

Because "1" is less than 5, the 5 remains the same. $\boxed{7.2935}$

STUDY TIP

When rounding, look to the right of the specified decimal place and use that digit (do not round that digit first). 5.23491 rounded to two decimal places is 5.23 (do not round the 4 to a 5 first).

YOUR TURN Approximate -2.381865 to four decimal places by

a. truncation **b.** rounding

ANSWER

a. Truncation: -2.3818

b. Rounding: -2.3819

It is important to note that *rounding and truncation sometimes yield the same approximation* (Example 3), *but not always* (Example 2).

0.1.3 Order of Operations

Addition, subtraction, multiplication, and division are called arithmetic operations. The results of these operations are called the sum, difference, product, and quotient, respectively. These four operations are summarized in the following table.

OPERATION	NOTATION	RESULT
Addition	$a + b$	Sum
Subtraction	$a - b$	Difference
Multiplication	$a \cdot b$ or ab or $(a)(b)$	Product
Division	$\frac{a}{b}$ or a/b $(b \neq 0)$	Quotient (Ratio)

Since algebra involves *variables* such as x, the traditional multiplication sign $\times$ is not used. Three alternatives are shown in the preceding table. Similarly, the arithmetic sign for division $\div$ is often represented by vertical or slanted fractions.

The symbol $=$ is called the **equal sign** and is pronounced "equals" or "is." It implies that the expression on one side of the equal sign is equivalent to (has the same value as) the expression on the other side of the equal sign.

WORDS	MATH
The sum of seven and eleven equals eighteen:	$7 + 11 = 18$
Three times five is fifteen:	$3 \cdot 5 = 15$
Four times six equals twenty-four:	$4(6) = 24$
Eight divided by two is four:	$\frac{8}{2} = 4$
Three subtracted from five is two:	$5 - 3 = 2$

When evaluating expressions involving real numbers, it is important to remember the correct *order of operations*. For example, how do we simplify the expression $3 + 2 \cdot 5$? Do we multiply first and then add, or do we add first and then multiply? In mathematics, conventional order implies multiplication first and then addition: $3 + 2 \cdot 5 = 3 + 10 = 13$. Parentheses imply grouping of terms, and the necessary operations should always be performed inside them first. If there are nested parentheses, always start with the innermost parentheses and work your way out. Within parentheses follow the conventional order of operations. Exponents are an important part of the order of operations and will be discussed in Section 0.2.

ORDER OF OPERATIONS

1. Start with the innermost parentheses (grouping symbols) and work outward.
2. Perform all indicated multiplications and divisions, working from left to right.
3. Perform all additions and subtractions, working from left to right.

EXAMPLE 4 **Simplifying Expressions Using the Correct Order of Operations**

Simplify the expressions.

a. $4 + 3 \cdot 2 - 7 \cdot 5 + 6$ **b.** $\dfrac{7 - 6}{2 \cdot 3 + 8}$

Solution (a):

Perform multiplication first. $4 + \underbrace{3 \cdot 2}_{6} - \underbrace{7 \cdot 5}_{35} + 6$

Then perform the indicated additions and subtractions. $= 4 + 6 - 35 + 6 = \boxed{-19}$

Solution (b):

The numerator and the denominator are similar to expressions in parentheses. Simplify these separately first, following the correct order of operations.

Perform multiplication in the denominator first. $\dfrac{7 - 6}{\underbrace{2 \cdot 3}_{6} + 8}$

Then perform subtraction in the numerator and addition in the denominator. $= \dfrac{7 - 6}{6 + 8} = \boxed{\dfrac{1}{14}}$

YOUR TURN Simplify the expressions.

a. $-7 + 4 \cdot 5 - 2 \cdot 6 + 9$ **b.** $\dfrac{9 - 6}{2 \cdot 5 + 6}$

0.1.3 SKILL

Simplify expressions and evaluate algebraic expressions using the correct order of operations.

0.1.3 CONCEPTUAL

Learn the order of operations for real numbers.

[CONCEPT CHECK]

TRUE OR FALSE In Example 4(*a*), we could have also started with adding 4 + 3.

ANSWER False

ANSWER

a. 10 **b.** $\frac{3}{16}$

Parentheses ( ) and brackets [ ] are the typical notations for grouping and are often used interchangeably. When nesting (groups within groups), use parentheses on the innermost and then brackets on the outermost.

EXAMPLE 5 Simplifying Expressions That Involve Grouping Signs Using the Correct Order of Operations

Simplify the expression $3[5 \cdot (4 - 2) - 2 \cdot 7]$.

Solution:

Simplify the inner parentheses. $3[5 \cdot (4 - 2) - 2 \cdot 7] = 3[5 \cdot 2 - 2 \cdot 7]$

Inside the brackets, perform the multiplication $5 \cdot 2 = 10$ and $2 \cdot 7 = 14$. $= 3[10 - 14]$

Inside the brackets, perform the subtraction. $= 3[-4]$

Multiply. $= \boxed{-12}$

YOUR TURN Simplify the expression $2[-3 \cdot (13 - 5) + 4 \cdot 3]$.

ANSWER

-24

Algebraic Expressions

Everything discussed until now has involved real numbers (explicitly). In algebra, however, numbers are often represented by letters (such as x and y), which are called **variables**. A **constant** is a fixed (known) number such as 5. A **coefficient** is the constant that is multiplied by a variable. Quantities within the *algebraic expression* that are separated by addition or subtraction are referred to as **terms**.

DEFINITION Algebraic Expression

An **algebraic expression** is the combination of variables and constants using basic operations such as addition, subtraction, multiplication, and division. Each term is separated by addition or subtraction.

Algebraic Expression	Variable Term	Constant Term	Coefficient
$5x + 3$	$5x$	3	5

When we know the value of the variables, we can **evaluate an algebraic expression** using the **substitution principle:**

Algebraic expression: $5x + 3$
Value of the variable: $x = 2$
Substitute $x = 2$: $5(2) + 3 = 10 + 3 = 13$

EXAMPLE 6 Evaluating Algebraic Expressions

Evaluate the algebraic expression $7x + 2$ for $x = 3$.

Solution:

Start with the algebraic expression. $7x + 2$

Substitute $x = 3$. $7(3) + 2$

Perform the multiplication. $= 21 + 2$

Perform the addition. $= \boxed{23}$

YOUR TURN Evaluate the algebraic expression $6y + 4$ for $y = 2$.

ANSWER

16

In Example 6, the value for the variable was specified in order for us to evaluate the algebraic expression. What if the value of the variable is not specified; can we simplify an expression like $3(2x - 5y)$? In this case, we cannot subtract $5y$ from $2x$. Instead, we rely on the basic *properties of real numbers*, or the *basic rules of algebra*.

0.1.4 SKILL

Apply properties of real numbers and basic rules of algebra in simplifying and evaluating expressions.

0.1.4 CONCEPTUAL

Know and understand the basic properties of real numbers and the basic rules of algebra.

0.1.4 Properties of Real Numbers

You probably already know many properties of real numbers. For example, if you add up four numbers, it does not matter in which order you add them. If you multiply five numbers, it does not matter in what order you multiply them. If you add 0 to a real number or multiply a real number by 1, the result yields the original real number. **Basic properties of real numbers** are summarized in the following table. Because these properties are true for variables and algebraic expressions, these properties are often called the **basic rules of algebra**.

PROPERTIES OF REAL NUMBERS (BASIC RULES OF ALGEBRA)

NAME	DESCRIPTION	MATH (LET *a*, *b*, AND *c* EACH BE ANY REAL NUMBER)	EXAMPLE
Commutative property of addition	Two real numbers can be added in any order.	$a + b = b + a$	$3x + 5 = 5 + 3x$
Commutative property of multiplication	Two real numbers can be multiplied in any order.	$ab = ba$	$y \cdot 3 = 3y$
Associative property of addition	When three real numbers are added, it does not matter which two numbers are added first.	$(a + b) + c = a + (b + c)$	$(x + 5) + 7 = x + (5 + 7)$
Associative property of multiplication	When three real numbers are multiplied, it does not matter which two numbers are multiplied first.	$(ab)c = a(bc)$	$(-3x)y = -3(xy)$
Distributive property	Multiplication is distributed over *all* the terms of the sums or differences within the parentheses.	$a(b + c) = ab + ac$ $a(b - c) = ab - ac$	$5(x + 2) = 5x + 10$ $5(x - 2) = 5x - 10$
Additive identity property	Adding zero to any real number yields the same real number.	$a + 0 = a$ $0 + a = a$	$7y + 0 = 7y$
Multiplicative identity property	Multiplying any real number by 1 yields the same real number.	$a \cdot 1 = a$ $1 \cdot a = a$	$(8x)(1) = 8x$
Additive inverse property	The sum of a real number and its additive inverse (opposite) is zero.	$a + (-a) = 0$	$4x + (-4x) = 0$
Multiplicative inverse property	The product of a nonzero real number and its multiplicative inverse (reciprocal) is 1.	$a \cdot \frac{1}{a} = 1 \quad a \neq 0$	$(x + 2) \cdot \left(\frac{1}{x + 2}\right) = 1$ $x \neq -2$

The properties in the previous table govern addition and multiplication. Subtraction can be defined in terms of addition of the *additive inverse*, and division can be defined in terms of multiplication by *the multiplicative inverse (reciprocal)*.

SUBTRACTION AND DIVISION

Let a and b be real numbers.

	MATH	TYPE OF INVERSE	WORDS
Subtraction	$a - b = a + (-b)$	$-b$ is the **additive inverse** or **opposite** of b	Subtracting a real number is equal to adding its opposite.
Division	$a \div b = a \cdot \frac{1}{b}$ $b \neq 0$	$\frac{1}{b}$ is the **multiplicative inverse** or **reciprocal** of b	Dividing by a real number is equal to multiplying by its reciprocal.